Probability of Electron Ion $\text{Li}^{2+}$ In Momentum Space At $n \leq 3$

Bambang Supriadi¹, Nilam Cahya Kusumaningtyas¹, Ana Zuyyina Ulfah¹, Safina Aulia Sani²*, Rike Dwi Wulandari¹

¹ Departemen Physics Education, Faculty of Teacher Training and Education, University of Jember, Indonesia

Abstract: Lithium ions ($\text{Li}^{2+}$) is a particle that has a single electron and is formed because lithium metal loses 2 of its 3 valence electrons. Probability is the square modulus of a wave function. Electron probability is a description of the discovery of electrons in space. This study aims to determine the probability value of electron momentum of Lithium ion ($\text{Li}^{2+}$). This type of research is non-experimental research in the form of developing existing theories with theoretical research methods. When viewed from the main orbital quantum value, the greater the probability value or the easier the electron is found at a larger quantum number. When viewed from the level of the probability interval, the greater the probability interval, the greater the main quantum value. In accordance with the theory which states that the large main quantum number and orbital quantum result in the probability value of Lithium ions getting bigger.

Keywords: Lithium Ion; Momentum Space; Probability

Introduction

Developments in physics are typically observed through experiments, but emerging theories derived from profound ideas can also provide new insight and transform our understanding of the universe. A prime example of such a development is the theory of quantum mechanics, which explains the relationship between particles and waves (Damayanti et al., 2019). The quantum mechanical theory of atomic structure is one of the fundamental concepts in quantum physics, and indeed in physics in general (Manik et al., 2022). This theory allows us to understand the structure of all matter and opens the veil of mystery to what it actually consists of. The results based on this theory are quite unexpected (Gopi et al., 2022). Quantum mechanical theory emerged to explain the relationship between waves and particles in which it examines things that are in the microscopic scope (Utami et al., 2019). Quantum mechanical theory explains the interrelationship of waves and particles in analyzing things in a microscopic scope such as atoms (Supriadi et al., 2023). Alkali group atoms such as are the lightest solid elements with mass numbers 6 (92.5 percent) and 7 (7.5 percent). The nucleus $\text{Li}^{2+}$ oscillates in instability because the two stable isotopes $\text{Li}^{2+}$ have the lowest binding energy per nucleon of any stable nuclide, even though the nucleus is very light $\text{Li}^{2+}$, although the nucleus is very light, it is less abundant in the solar system than 25 of the first 32 chemical elements due to its relative nuclear instability.

The quantum theory of the $\text{Li}^{2+}$ atom was not deduced directly from the Schrödinger Equation. Rather, its energy levels and wave functions were derived based on those of the Hydrogen atom. $\text{Li}^{2+}$ serves as an example of an atom possessing multiple electrons (Men & Setianto, 2017). Ion $\text{Li}^{2+}$ are particles that have a single electron and are formed because lithium metal loses 2 of its 3 valence electrons. Lithium ions have an atomic number ($Z$ of 3 which states that there are 3 protons in the ion). Utilization of ions $\text{Li}^{2+}$ this is found in electronic components commonly used for rechargeable batteries or often referred to as ion batteries. $\text{Li}^{2+}$. Lithium ions are cations (positive ions) that can depolarize the neuronal membrane as sodium ions do in depolarization phase of action potential but this requires that the membrane conductance for lithium

How to Cite:
is at least an order of magnitude higher than the membrane conductance for sodium ions to induce a significant depolarization (Thiruvengadam, 2001) which would be impossible since lithium and sodium ions have the same conductance through channels as they are selective for both cations by the same degree (Qaswali, 2020). 3Li\(^{2+}\) ion batteries are classified into two types, including primary cell batteries and secondary cell batteries. The method used to determine the existence of visualization of atomic electrons is the possibility of opportunity that occurs in an atom (probability).

Probability is the square modulus of a wave function. Electron probability is a description of the electron found in space (Kharisamwati and Supriadi, 2020). If in a certain area in space the value of electron probability is high then the electron is likely to be large and vice versa if the electron probability value is small it is unlikely that electrons are found in that area.

The Schrödinger equation is a second-order differential equation discovered by Erwin Schrödinger. This equation should be expressed objectively, using clear and concise language with logical connections between statements. It is important to follow established academic structure and formatting conventions, employ a formal register, and use precise subject-specific vocabulary. It provides insights into the behavior of a particle, such as determining its wave function and associated probability (Suprami et al., 2018; Wardani, 2019). The Schrödinger equation in advanced mechanics functions to provide wave information on a particle in the form of a second-order differential equation (Utami et al., 2019). The Schrödinger equation produces a complex analytic solution in the form of a single-valued wave function, continuous, and infinite. In this study, the solution of the Schrödinger equation of the lithium atom is studied as follows:

**Wave Function Equation in Position Space**

\[
\psi_{n,l,m}(r, \theta, \phi) = \left( \frac{2}{n_a a_0} \right)^{\frac{3}{2}} \left( \frac{(n-l-1)!}{(n+l)!} \right)^{\frac{1}{2}} 2^{l+\frac{1}{2}} \left( \frac{2\pi}{r} \right)^{l+\frac{1}{2}} \frac{\sqrt{2\pi}}{2^{l+\frac{1}{2}}} e^{i\pm im\phi} p_l^m \cos \theta \\
\text{if } \gamma = \frac{Z}{n_a a_0} \text{ then} \\
\psi_{n,l,m}(r, \theta, \phi) = \left( \frac{2\gamma}{n_a a_0} \right)^{\frac{3}{2}} \frac{\sqrt{2\pi}}{2^{l+\frac{1}{2}}} e^{i\pm im\phi} p_l^m \\
\psi_{n,l,m}(r, \theta, \phi) = \left( \frac{2\gamma}{n_a a_0} \right)^{\frac{3}{2}} \frac{\sqrt{2\pi}}{2^{l+\frac{1}{2}}} e^{i\pm im\phi} p_l^m \\
(2) \text{ (3)}
\]

**Wave Function Equation in Momentum Space**

\[
\varphi = \frac{1}{(2\pi)^{\frac{3}{2}}} \frac{2l+1}{(l+m)!} \frac{1}{(l+|m|)!} p_l^m \\
\cos \theta \frac{\pi l^l}{(\gamma h)^{\frac{3}{2}}} 2^{l+1} \frac{1}{2^{l+\frac{1}{2}}} \frac{\sqrt{2\pi}}{2^{l+\frac{1}{2}}} e^{i\pm im\phi} p_l^m \\
(4)
\]

**Radial Function Equation**

\[
F_{nl} = \frac{2^{l+\frac{3}{2}} \left( \frac{2\gamma}{n_a a_0} \right)^{\frac{3}{2}} \frac{\sqrt{2\pi}}{2^{l+\frac{1}{2}}} e^{i\pm im\phi} p_l^m \times}{\sqrt{n_a a_0} \left( \frac{(n-l-1)!}{(n+l)!} \right)^{\frac{1}{2}} 2^{l+\frac{1}{2}}} \\
\text{if } \gamma = \frac{Z}{n_a a_0} \text{ then} \\
F_{nl} = \frac{2^{l+\frac{3}{2}} \left( \frac{2\gamma}{n_a a_0} \right)^{\frac{3}{2}} \frac{\sqrt{2\pi}}{2^{l+\frac{1}{2}}} e^{i\pm im\phi} p_l^m \times}{\sqrt{n_a a_0} \left( \frac{(n-l-1)!}{(n+l)!} \right)^{\frac{1}{2}} 2^{l+\frac{1}{2}}} \\
(5)
\]

Such atomic wave functions are of physical interest as probability values increase in length. Probability is how big the chance is the possibility of finding the electron location of the atom. The wave function refers to the radial wave function of the atom to find the probability at the electron position which requires a partial wave function of the radial part only (Saputra, 2019; Naimah, 2019). The sentence can be summarized as follows: “All matter appears solid due to the interaction between atoms. Substances have a solid and hard consistency only because of the attraction or repulsion between atoms. This ensures the density and hardness of the chemical crystal lattice, which contains all substances. However, when environmental temperature conditions change, the bond between atoms, namely their attraction and repulsion, can weaken, causing the crystal lattice to weaken and even collapse. This explains changes in the physical properties of materials when heated. Scientists have discovered another mysterious property of elementary particles. This is how the concept of uncertainty and wave function collapse arises in quantum physics (Konstantopoulos, 2022). Here is the wave function on the atom 3Li\(^{2+}\) in the equation:
Based on the wave function, the fact that the problem of single-electron lithium ions can be solved using the Schrodinger equation approach, makes researchers interested in studying it further. Therefore, this study will examine the probability of Lithium ions ($\text{Li}^{2+}$) in momentum space with quantum numbers ($n \leq 3$).

**Method**

The type of research used in this study is non-experimental research the form of developing existing theories with theoretical research methods. This research was conducted in the even semester of the 2022/2023 academic year in Physics Education, University of Jember. A step is in this research preparation, theory development, validation of theories, results or data collection, discussion and conclusions. The research process is shown in Figure 1.

![Figure 1. The research process](image)

The first step is to collect some literature for reference and information in this research. The references used include relevant national and international journals with the Schrödinger Equation on ions and the probability value of electron positions in ions $\text{Li}^{2+}$.

The second step in this research is the development of theories carried out by researchers by further developing previous theories derived from some literature and studies related to previous research. Most of the previous theories contained in the literature related to wave functions for hydrogen atoms only and their isotopes. In this study, the theory developed is the probability value of the electron location of the ion $\text{Li}^{2+}$ validation of theory development. To express the probability function in space on the electron ion $\text{Li}^{2+}$ it can be expressed in the form of radial momentum function as follow (Formula 6).

$$F_{n1}(p) = \left(\frac{2}{\pi} \right)^{l} \frac{(n - l - 1)!^{\frac{1}{2}}}{n^{l+2}} \times \frac{n^{2}l!l^{l+2}}{n!^{2}}$$ \quad (6)

The function describe is a radial wave function specific to hydrogen atoms, and it’s express in atomic units for momentum or scaled by $p_{0}$ (Bransden, 1983). According to the equation provided above, when the atomic number of the lithium ion $(z)$ is 2, the radial momentum wave for the lithium ion can be described as follow (Formula 7).

$$F_{n1} = \frac{2^{l+2}}{n!^{2}} \times \frac{n^{2}l!l^{l+2}}{n!^{2}} \times \left(\frac{n^{2}p^{2} - 9p_{0}^{2}}{n!^{2}}\right)$$ \quad (7)

The third stage of this research validated the probability values of electron positions in ions $\text{Li}^{2+}$ in the form of existing literature. The probability equation for determining the presence of particles within a space is as follows:

$$P_{(x,t)} = \int_{x_{1}}^{x_{2}} [\Psi(x,t)]^{2} dx$$

$$P_{(x,t)} = \int_{x_{1}}^{x_{2}} [\Psi(x,t)]^{2} dx$$ \quad (Supriadi et al., 2023).

In the cases of 3 dimensions, the probability equation becomes as follows:

$$P_{(x,t)} = \int_{r_{1}}^{r_{2}} [\Psi(r,t)]^{2} dr$$

To express the probability of particles describe by $\Psi(r,t)$ being between $r$ and $r + dr$ at a specific time $t$ (Supriadi et al., 2022; Yusron et al., 2007). Furthermore, there are results of data collection, namely researchers performing calculations to determine the probability value of electron positions in atomic $\text{Li}^{2+}$ quantum number $n \leq 3$ and simulation of the radial probability density curve of ions $\text{Li}^{2+}$ in quantum number $n \leq 3$ with Matlab 2018 software. The final stage of this research is a discussion in which the researcher discusses in detail the information about the size of the probability value of the location of electrons in ions $\text{Li}^{2+}$ in quantum numbers $n \leq 3$.

**Result and Discussion**

Based numerical calculations using on 2018, the data obtained to find electrons on ions $\text{Li}^{2+}$ for $n \leq 3$ with momentum space review is as follows:
There are three cusps simultaneously.

When viewed from the main orbital quantum value, the greater the probability value or the easier the electron is found at a larger quantum number. This can be proven from the results of the data on the value of \( n = 1 \), \( l = 0 \) value \( 1p_0 \), is 0.0979; \( n = 2 \), \( l = 0 \) value \( 1p_0 \), is 0.7802; \( n = 3 \), \( l = 0 \) value \( 1p_0 \), is 0.8092 as well as the next interval. When viewed from the level of the probability interval, the greater the probability interval, the greater the main quantum value. This can be proven from the data results at the lowest interval, which is \( 1p_0 \), is 0.0979 and at the highest interval, namely \( 9p_0 \), is 0.9991; \( n = 1 \), \( l = 0 \) at \( n = 1 \), \( l = 0 \) is 0.9938 at \( n = 1 \), \( l = 0 \). At \( n = 2 \), \( l = 0 \) interval \( 1p_0 \), is 0.7802 and the highest interval \( 9p_0 \), is 0.9991, as well as the next main quantum value. So, if the momentum calculation is more accurate, the less accurate the calculation of the lithium atom in its position space. This is in accordance with the Heisenberg uncertainty equation which states that the relationship between momentum and position uncertainty is as follows:

\[
\Delta P_x \Delta x \geq \frac{h}{2} \text{ with } h = \frac{\hbar}{2\pi} = 1.054.10^{-34} Js
\] (10)

Where, if measurements from momentum space are made with uncertainty \( \Delta P_x \) and \( \Delta x \), measurement of the particle’s position is made with, then the product of the two uncertainties cannot be smaller than \( \frac{h}{2} \). So it is physically impossible to measure exactly between atoms or particles in momentum space and position space simultaneously.

When viewed from each state separately, Figure 1 shows that in the \( n = 1 \), \( l = 0 \) state, there is only one peak point. For the \( n = 2 \), \( l = 0 \) state, there are two peak points. Furthermore, in the \( n = 3 \) state, there are three cusp points formed. This number of cusps reflects the various opportunities to find electrons in each state based on their orbital shape. It has previously been explained that each quantum number has a different orbital shape.

In addition, it is also known that the highest number of peak points at each main quantum number \( n \) occurs at state \( l = 0 \). This is due to the fact that in this state, sub-skin 1 is the closest to the skin, so the distance between the electron and the atomic nucleus is the smallest. Therefore, the presence of electrons in this state is easier to find.

Based on the spatial distribution graph and the momentum distribution of the research results (Frank, 2023), it can be said that Figure 1 graph of probability result data on ions \( _3Li^{2+} \) for \( n \leq 3 \) Using Matlab 2018 is valid. This can be shown in the results of \( n = 11 = 0 \) there is 1 peak, \( n = 21 = 0 \) there are 2 peaks and \( n = 31 = 0 \) there are 3 peaks.

**Table 1. Result of probability values on ions \( _3Li^{2+} \) for \( n \leq 3 \)**

<table>
<thead>
<tr>
<th></th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
<th>( n = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_0 )</td>
<td>( 1p_0 )</td>
<td>( 2p_0 )</td>
<td>( 3p_0 )</td>
</tr>
<tr>
<td>( l = 0 )</td>
<td>0.0979</td>
<td>0.4282</td>
<td>0.7122</td>
</tr>
<tr>
<td>( l = 1 )</td>
<td>0.3109</td>
<td>0.8539</td>
<td>0.9298</td>
</tr>
<tr>
<td>( l = 2 )</td>
<td>0.8092</td>
<td>0.9516</td>
<td>0.9784</td>
</tr>
</tbody>
</table>

**Figure 2.** Graph of probability result data on ions \( _3Li^{2+} \) for \( n \leq 3 \) using Matlab 2018

**Figure 3.** Spatial distribution graph (Frank., 2023)

**Figure 4.** Momentum distribution graph (Frank., 2023)
Relevant studies state that the quantum Hilbert space of a harmonic oscillator interacting with a monochromatic wave is divided into the quantum resonance cells. The dynamical manifestation of such a division is the property of localization over the quantum resonance cells. Approach to the quantum problem is based on the quasienergy states. It was shown that the Hilbert space of the quantum system is reasonable to separate into quantum resonance cells similar to the resonance cells in the classical phase space and to measure the average quantum probability in each cell. This technique will allow one to simplify the measurement of the probability distribution, because in this case it is not necessary to measure the probability at a fixed oscillator state (Berman et al., 2000), and Optical frequency conversion via nonlinear processes is well-established and may be used to obtain more desirable wavelengths. Quantum frequency conversion (QFC) of singlephotons has been demonstrated using atomic vapors, photonic crystals such as lithium niobate (LN) (Sivers et al., 2017).

**Conclusion**

Based on the calculation of the Probability of Ion Electron Momentum $3\text{Li}^{3+}$ in Momentum Space At $n \leq 3$, when viewed from the main orbital quantum value, the greater the probability value or the easier the electron is found at a larger quantum number, and when viewed from the probability interval level, the greater the probability interval, the greater the main quantum value. In addition, when viewed from the probability graph, valid graph data is produced.

**Acknowledgments**

Our research team would like to thank all participants who have been involved in this study. In addition, our research team would like to thank all reviewers and the JPPIPA editorial team who have given us the opportunity and assisted in the publication process of the article manuscripts that we have compiled.

**Author Contributions**

Bambang Supriadi contributes to conceptualizing the research idea, and guiding the writing process. Nilam Cahya Kusumaningtyas contributes to probability and momentum space calculating. Ana Zuyyina Ulfah and Rike Dwi Wulandari contributed to article writing, reviewed and edited. Safina Aulia Sani contributes to article writing and graph calculation using Matlab.

**Funding**

This research was self-funded by the author.

**Conflicts of Interest**

The authors declare no conflicts of interest relevant to the publication of this manuscript.

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